

Credit and annuity, example

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Lets start with simple example. You want to borrow money from the bank, lets say 50 000, or any other amount, whatever it is, we will call it A_0 . For five years, or any other period, we'll call it T . You promise to pay monthly equal amounts to return the debt, these amounts are called annuity (A series of fixed payments paid at regular intervals over the specified period of the annuity). That means you'll pay in 60 equal amounts (T times 12). Bank clerk says that you should pay yearly interest rates of, let's say 9.9% (lets call that percentage P). How do they calculate annuity? Here's how:

Lets find a number with which I need to multiply my debt to get the debt after a year, if I'm not returning money at all, then we'll find for month. It's easy, that's $1 + P/100$. If I owe 100 now, after a year I'll owe $100 \times 1.099 = 109.9$. Why do we need monthly interest rate? Because you want to pay every month, not every year. OK, so what's the problem you say, *I just divide 9.9 with 12 and there it is!* NO! No, no, no, big mistake! Why not? Here's the problem, you need a rate that will accumulate during the year to 9.9%. So, monthly percentage times monthly percentage times monthly ... and so on (for twelve months) needs to be 9.9%. Lets name monthly percentage as r . So $r \times r \times r \dots r = r^{12} = 1 + P/100$. Thus, the monthly rate will be

$$r = (1 + \frac{P}{100})^{1/12} \quad (1)$$

OK, now we have r for month. If you borrow 100, after one month you'll owe $100 \times r$, after two months $100 \times r^2$ and after twelve months $100 \times r^{12}$ which is exactly $100 \times (1 + P/100)$. That's, of course if you don't pay anything (!) to the bank during that period. Let's calculate that annuity already 'cause this is getting boring. OK, lets see. You are paying your rate R every month.

month	you owe	you pay
0	A_0	0
1	$A_0 r - R$	R
2	$(A_0 r - R)r - R$	R
3	$((A_0 r - R)r - R)r - R$	R
4	$((((A_0 r - R)r - R)r - R)r - R)$	R
	...	
60	see text	R

So you owe every time less money, but bank wants inter-

est on their money which you haven't returned yet. At the beginning you get the A_0 and don't pay them anything. After a month, they say, *You owe us $A_0 r$ because of monthly interest rates*, and you pay them your R for that month. So they take that R off of your debt. At next month, they want their interest for the money they gave you (those $A_0 r - R$) and you pay them your R this month also, and so on ... Now, if you look better you owe them after sixty months

$$A_0 r^{60} - R(r^{59} + r^{58} + \dots + r + 1) \quad (2)$$

But, wait, you should repay your debt at sixty months. So that must be zero. Great, there's an equation that'll tell us something about R since we know all other values already.

$$A_0 r^{60} - R(r^{59} + r^{58} + \dots + r + 1) = 0 \quad (3)$$

Now, there's an easy formula for things like

$$1 + r + r^2 + r^3 + \dots + r^n \quad (4)$$

It says this:

$$1 + r + r^2 + r^3 + \dots + r^n = \frac{r^{n+1} - 1}{r - 1} \quad (5)$$

So if m is the number of months you need to pay your debt, then we get:

$$A_0 r^m - R \sum_{i=0}^{m-1} r^i = A_0 r^m - R \frac{r^m - 1}{r - 1} = 0 \quad (6)$$

Now, we solve for R , and get:

$$R = A_0 \frac{r^m (r - 1)}{r^m - 1} \quad (7)$$

By using that formula, we get that for 50 000 repayed in 60 months with 9.9 % interest, you should pay every month:

$$R = 50000 \times 0.020990 \approx 1049.53 \quad (8)$$